Appendices for:

New pseudo-dynamic analysis of two-layered cohesive-friction soil slope and its numerical validation

Suman Hazari, Sima Ghosh, Richi Prasad Sharma

Department of Civil Engineering, National Instituteof Technology, Agartala 799046, India

*Corresponding author. E-mail: sima.civil@nita.ac.in

Appendix

The detail expressions of I_1, I_2, I_3 and I_4 are as follows:

$$I_1 = \begin{bmatrix} \{y_{s2}\sinh(k_{s2}z) + y_{s4}\cosh(k_{s2}z)\}\left\{\sin\left(2\pi\frac{t}{T} - k_{s1}z\right) - \sin\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} + \\ \{y_{s3}\sinh(k_{s2}z) + y_{s1}\cosh(k_{s2}z)\}\left\{\cos\left(2\pi\frac{t}{T} - k_{s1}z\right) + \cos\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} + \\ \{y_{s3}\cosh(k_{s2}z) + y_{s1}\sinh(k_{s2}z)\}\left\{\cos\left(2\pi\frac{t}{T} - k_{s1}z\right) - \cos\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} + \\ \{y_{s2}\cosh(k_{s2}z) + y_{s4}\sinh(k_{s2}z)\}\left\{\sin\left(2\pi\frac{t}{T} - k_{s1}z\right) + \sin\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} \end{bmatrix},$$

$$I_2 = \begin{bmatrix} \{y_{s2}\sinh(k_{s2}z) - y_{s4}\cosh(k_{s2}z)\}\left\{\sin\left(2\pi\frac{t}{T} - k_{s1}z\right) - \sin\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} - \\ \{y_{s3}\sinh(k_{s2}z) - y_{s1}\cosh(k_{s2}z)\}\left\{\cos\left(2\pi\frac{t}{T} - k_{s1}z\right) + \cos\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} - \\ \{y_{s3}\cosh(k_{s2}z) - y_{s1}\sinh(k_{s2}z)\}\left\{\cos\left(2\pi\frac{t}{T} - k_{s1}z\right) - \cos\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} + \\ \{y_{s2}\cosh(k_{s2}z) - y_{s4}\sinh(k_{s2}z)\}\left\{\sin\left(2\pi\frac{t}{T} - k_{s1}z\right) + \sin\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} \end{bmatrix},$$

$$I_3 = \begin{bmatrix} -\{y_{s2}\sinh(k_{s2}z) - y_{s4}\cosh(k_{s2}z)\}\left\{\cos\left(2\pi\frac{t}{T} - k_{s1}z\right) - \cos\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} + \\ \{y_{s3}\sinh(k_{s2}z) - y_{s1}\cosh(k_{s2}z)\}\left\{\sin\left(2\pi\frac{t}{T} - k_{s1}z\right) + \sin\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} + \\ \{y_{s3}\cosh(k_{s2}z) - y_{s1}\sinh(k_{s2}z)\}\left\{\sin\left(2\pi\frac{t}{T} - k_{s1}z\right) - \sin\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} - \\ \{y_{s2}\cosh(k_{s2}z) - y_{s4}\sinh(k_{s2}z)\}\left\{\cos\left(2\pi\frac{t}{T} - k_{s1}z\right) + \cos\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} \end{bmatrix},$$

$$I_4 = \begin{bmatrix} -\{y_{s2}\sinh(k_{s2}z) - y_{s4}\cosh(k_{s2}z)\}\left\{\cos\left(2\pi\frac{t}{T} - k_{s1}z\right) - \cos\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} - \\ \{y_{s3}\sinh(k_{s2}z) - y_{s1}\cosh(k_{s2}z)\}\left\{\sin\left(2\pi\frac{t}{T} - k_{s1}z\right) + \sin\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} - \\ \{y_{s3}\cosh(k_{s2}z) - y_{s1}\sinh(k_{s2}z)\}\left\{\sin\left(2\pi\frac{t}{T} - k_{s1}z\right) - \sin\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} + \\ \{y_{s2}\cosh(k_{s2}z) - y_{s4}\sinh(k_{s2}z)\}\left\{\cos\left(2\pi\frac{t}{T} - k_{s1}z\right) + \cos\left(2\pi\frac{t}{T} + k_{s1}z\right)\right\} \end{bmatrix}.$$

Reffering to Fig. A1, the expressions of $w_{ODPE}, w_{BEE_1}, w_{ODD_1}, w_{OAME}, w_{ACJ}$ and w_{OD_1J} are given by

$$w_{ODPEO} = \frac{r_1^2 - r_0^2}{4\tan\phi} = \gamma_1 r_0^2\left[\frac{e^{2\theta_1\tan\phi} - 1}{4\tan\phi}\right],$$

$$w_{BEE_1} = 0.5\gamma\left[\left[\left\{\frac{H_1}{\sin\beta} - \left[\left\{\frac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)} - b_s\right\}\frac{\sin(\theta_1+\theta_b)}{\sin(\theta_1+\theta_b+\beta)}\right]\right]\left\{\frac{H_1}{\sin(\theta_1+\theta_b)}\right\}\sin(\theta_1+\theta_b+\beta)\right]$$

$$, w_{ODD_1} = 0.5\gamma r_0^2\left[\frac{\sin\theta_1\sin\theta_b}{\sin(\theta_1+\theta_b)}\right],$$

$$w_{CD_1E_1} = 0.5\gamma\left[\left\{\frac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)} - b_s\right\}\left\{\frac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)} - b_s\right\}\left\{\frac{\sin(\theta_1+\theta_b)}{\sin(\theta_1+\theta_b+\beta)}\right\}\right],$$

$$w_{OAMEO} = \frac{r_2^2 - r_1^2}{4\tan\phi} = \gamma r_0^2\left[\frac{e^{2\theta_2\tan\phi} - 1}{4\tan\phi}\right],$$

$$w_{ACJ} = 0.5\gamma\left[\{H[\cot\beta - \cot(\theta_a+\theta_b)]\}^2\sin(\theta_a+\theta_b)\frac{\sin\beta}{\sin(\theta_a+\theta_b+\beta)}\right],$$

$$w_{OD_1J} = 0.5\gamma\left[\left\{H\left[\cot\beta - \cot(\theta_a+\theta_b)\right] - \frac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)} + b_s\right\}^2 \frac{\sin(\theta_a+\theta_b)}{\sin\theta_2}\sin(\theta_1+\theta_b)\right].$$

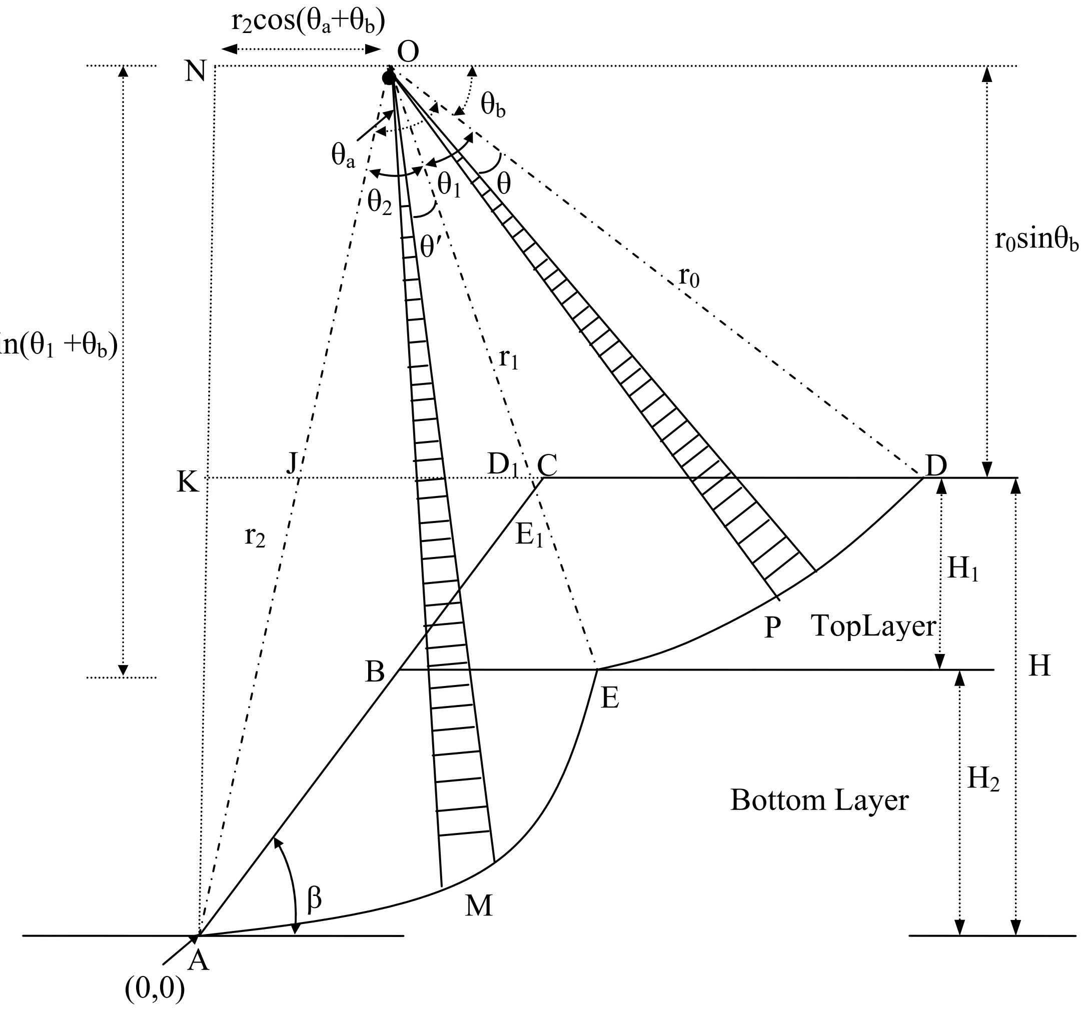

Fig. A1 Geometry for the calculation of the center of gravity and moment of forces.

Referring to Fig. A1, the detailed expressions of $M_{ODPE}, M_{BEE_1}, M_{ODD_1}$, M_{OAME}, M_{ACJ}, $M_{OD_1J}, M'_{ODPE}, M'_{BEE_1}, M'_{ODD_1}$, M'_{OAME}, M'_{ACJ}, and M'_{OD_1J} are as follows:

$$M_{ODPEO} = \frac{\gamma r_0^3}{3(1+9\tan^2\phi)}\left[e^{3\theta_1 \tan\phi}\{3\tan\phi\cos(\theta_1+\theta_b)+\sin(\theta_1+\theta_b)\}-3\tan\phi\cos\theta_b-\sin\theta_b\right]$$

,

$$M_{BEE_1} = 0.5\gamma\left[\left\{\frac{H_1}{\sin\beta}-\frac{\sin(\theta_1+\theta_b)}{\sin(\theta_1+\theta_b+\beta)}\left(\frac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)}-b_s\right)\right\}\left\{\frac{H_1}{\sin(\theta_1+\theta_b)}\right\}\times\sin(\theta_1+\theta_b+\beta)\right]$$
$$\left[\left\{\frac{H_2}{\sin\beta}+\frac{2}{3}\frac{\sin(\theta_1+\theta_b)}{\sin(\theta_1+\theta_b+\beta)}\left(\frac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)}-b_s\right)\right\}\cos\beta+\cos(\beta+\partial'')-r_2\cos(\theta_1+\theta_b)\right]$$

,

$$M_{ODD_1} = \frac{2}{3}\gamma r_0^3\frac{\sin\theta_1\sin\theta_b\cos(\theta_b+\delta)}{4\sin\delta\sin(\theta_1+\theta_b)(\sin\theta_1\cos\theta_b+\cos\theta_1)},$$

$$M_{OAMEO} = \frac{\gamma r_0^3}{3(1+9\tan^2\phi)}\left[\begin{matrix}e^{3\theta_2\tan\phi}\{3\tan\phi\cos(\theta_1+\theta_2+\theta_b)+\sin(\theta_1+\theta_2+\theta_b)\}-3\tan\phi\cos(\theta_1+\theta_b)\\-\sin(\theta_1+\theta_b)\end{matrix}\right]$$

,

$$M_{ACJ} = 0.5\gamma\left[\frac{\sin(\theta_a+\theta_b)\sin\beta\{H[\cot\beta-\cot(\theta_a+\theta_b)]\}^2}{\sin(\theta_a+\theta_b+\beta)}\right]\left\{\begin{matrix}\frac{1}{3\sin\delta'}\{H\cot\beta-H\cot(\theta_a+\theta_b)\}\\\sin\beta\cos(\beta+\delta')-r_2\cos(\theta_a+\theta_b)\end{matrix}\right\}$$

$$,M_{OD_1J} = \frac{\gamma r_0}{6}\frac{I'^2\sin(\theta_a+\theta_b)\sin(\theta_1+\theta_b)\cos(\theta_1+\theta_b+\Delta)}{\sin\Delta(\sin\theta_1\cos\theta_b+\cos\theta_1)\sin\theta_2},$$

$$M_{CD_1E_1} = 0.5\gamma\left[\begin{matrix}\left\{\frac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)}-b_s\right\}\left\{\frac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)}-b_s\right\}\\\left\{\frac{\sin(\theta_1+\theta_b)}{\sin(\theta_1+\theta_b+\beta)}\right\}\end{matrix}\right]\left[\begin{matrix}H\cot\beta-r_2\cos(\theta_a+\theta_b)-\\\left(\frac{2}{3}\left\{\frac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)}-b_s\right\}\cos\Delta'\right)\end{matrix}\right],$$

$$M'_{ODPEO} = \frac{\gamma r_0^3}{3(1+9\tan^2\phi)}\left[e^{3\theta_1\tan\phi}\{3\tan\phi\sin(\theta_1+\theta_b)-\cos(\theta_1+\theta_b)\}-3\tan\phi\cos\theta_b+\cos\theta_b\right]$$

,

$$M'_{BEE_1} = 0.5\gamma\left[\left\{\frac{H_1}{\sin\beta}-\frac{\sin(\theta_1+\theta_b)}{\sin(\theta_1+\theta_b+\beta)}\left(\frac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)}-b_s\right)\right\}\left\{\frac{H_1}{\sin(\theta_1+\theta_b)}\right\}\sin(\theta_1+\theta_b+\beta)\right]$$
$$\left[H_1+r_0\sin\theta_b-\frac{H_1\sin^2\beta\sin(\theta_1+\theta_b)}{3\sin\partial''\sin(\theta_1+\theta_b+\beta)}\right]$$

,

$$M'_{ODD_1} = \frac{2}{3}\gamma r_0^3 \frac{\sin\theta_1 \sin\theta_b \sin(\theta_b + \delta)}{4\sin\delta \sin(\theta_1 + \theta_b)(\sin\theta_1 \cos\theta_b + \cos\theta_1)},$$

$$M'_{OAMEO} = \frac{\gamma r_0^3}{3(1+9\tan^2\phi)}\begin{bmatrix} e^{3\theta_2 \tan\phi}\{3\tan\phi \sin(\theta_1+\theta_2+\theta_b) - \cos(\theta_1+\theta_2+\theta_b)\} - 3\tan\phi\cos(\theta_1+\theta_b) \\ +\cos(\theta_1+\theta_b) \end{bmatrix}$$

,

$$M'_{ACJ} = 0.5\gamma\left[\frac{\sin(\theta_a+\theta_b)\sin\beta\{H[\cot\beta - \cot(\theta_a+\theta_b)]\}^2}{\sin(\theta_a+\theta_b+\beta)}\right]\left\{\begin{matrix} r_0\sin\theta_b + \dfrac{H}{3\sin\delta'} \\ \{H\cot\beta - H\cot(\theta_a+\theta_b)\} \\ \sin\beta\sin(\beta+\delta') - r_2\cos(\theta_a+\theta_b) \end{matrix}\right\}$$

$$, M'_{OD_1J} = \frac{\gamma r_0}{6}\frac{I'^2 \sin(\theta_a+\theta_b)\sin(\theta_1+\theta_b)\sin(\theta_1+\theta_b+\Delta)}{\sin\Delta(\sin\theta_1\cos\theta_b + \cos\theta_1)\sin\theta_2},$$

$$M'_{CD_1E_1} = 0.5\gamma\left[\begin{matrix} \left\{\dfrac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)} - b_s\right\}\left\{\dfrac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)} - b_s\right\} \\ \left\{\dfrac{\sin(\theta_1+\theta_b)}{\sin(\theta_1+\theta_b+\beta)}\right\} \end{matrix}\right]\left[\begin{matrix} r_0\sin\theta_b + \\ \left(\dfrac{2}{3}\left\{\dfrac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)} - b_s\right\}\sin\Delta'\right) \end{matrix}\right],$$

$$\delta = \tan^{-1}\left(\frac{\tan\theta_1 \tan\theta_b}{2\tan\theta_b + \tan\theta_1}\right),$$

$$\delta' = \cot^{-1}\left[\frac{2\sin(\theta_a+\theta_b)}{2\sin(\theta_a+\theta_b+\beta)\sin\beta\{H\cot\beta - h\cot(\theta_a+\theta_b)\}} - \cot\beta\right],$$

$$\Delta = \tan^{-1}\left[\frac{\sin\theta_1}{\tan\theta_b\left(\dfrac{2r_0 I'\tan\theta_b}{\sin\theta_1} - \cos\theta_1\cot\theta_b + \sin\theta_1\right)}\right],$$

$$\partial'' = \cot^{-1}\left[\frac{2\left\{\dfrac{H_1}{\sin\beta} - \left[\dfrac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)} - b_s\right]\left[\dfrac{\sin(\theta_1+\theta_b)}{\sin(\theta_1+\theta_b+\beta)}\right]\right\}}{\dfrac{H_1\sin(\theta_1+\theta_b)}{\sin(\theta_1+\theta_b+\beta)}} - \cot\beta\right],$$

$$\Delta' = \tan^{-1}\left[\frac{\sin(\theta_1+\theta_b)}{2\left\{\dfrac{\left[\dfrac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)}-b_s\right]}{\left[\dfrac{r_0\sin\theta_1}{\sin(\theta_1+\theta_b)}-b_s\right]\left[\dfrac{\sin(\theta_1+\theta_b)}{\sin(\theta_1+\theta_b+\beta)}\right]}\right\}-\cos(\theta_1+\theta_b)}\right].$$