Appendices for:

Free vibration analysis of functionally graded porous curved nanobeams on elastic foundation in hygro–thermal–magnetic environment

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Appendix A

The elements of the mass and stiffness matrices of Navier's solution are as follows:

$$\boldsymbol{K}=\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{12} & k_{22} & k_{23} \\ k_{13} & k_{23} & k_{33} \end{bmatrix}: k_{11}=\left(\mathsf{A}+\frac{2\mathsf{B}}{R_x}+\frac{\mathsf{F}}{R_x^2}\right)\lambda_m^2,\; k_{12}=-\left(\mathsf{B}+\frac{\mathsf{F}}{R_x}\right)\lambda_m^3-\left(\frac{\mathsf{A}}{R_x}+\frac{\mathsf{B}}{R_x^2}\right)\lambda_m,$$

$$k_{13}=-\left(\mathsf{B}^{\mathrm{b}}+\frac{\mathsf{F}^{\mathrm{b}}}{R_x}\right)\lambda_m^3-\left(\frac{\mathsf{A}}{R_x}+\frac{\mathsf{B}}{R_x^2}\right)\lambda_m, k_{22}=\mathsf{F}\lambda^4+\frac{2\mathsf{B}}{R_x}\lambda_m^2+\frac{\mathsf{A}}{R_x^2}+k_{\mathrm{w}}\left(1+\mu^2\theta_m^2\right),$$

$$+\left(k_{\mathrm{s}}+f^{\mathrm{HT}}+\eta hH_x\right)\lambda_m^2\left(1+\mu^2\lambda_m^2\right)$$

$$k_{23}=\mathsf{F}^{\mathrm{b}}\lambda_m^4+\left(\frac{\mathsf{B}}{R_x}+\frac{\mathsf{B}^{\mathrm{b}}}{R_x^2}\right)\lambda_m^2+\frac{\mathsf{A}}{R_x^2}+k_w\left(1+\mu^2\lambda_m^2\right)+\left(k_{\mathrm{s}}+f^{\mathrm{HT}}+\eta hH_x\right)\lambda_m^2\left(1+\mu^2\lambda_m^2\right),$$

$$k_{33}=\mathsf{H}\lambda_m^4+\left(\mathsf{A}^{\mathrm{b}}+\frac{2\mathsf{B}^{\mathrm{b}}}{R_x}\right)\lambda_m^4+\frac{\mathsf{A}}{R_x^2}+k_{\mathrm{w}}\left(1+\mu^2\lambda_m^2\right)+\left(k_{\mathrm{s}}+f^{\mathrm{HT}}+\eta hH_x\right)\lambda_m^2\left(1+\mu^2\lambda_m^2\right)$$

$$\boldsymbol{M}=\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}:\; m_{11}=I_1\left(1+\mu^2\lambda_m^2\right),\; m_{12}=-I_3\lambda\left(1+\mu^2\lambda_m^2\right),$$

$$m_{13}=-I_4\lambda_m\left(1+\mu^2\lambda_m^2\right),\; m_{22}=\left(I_2+I_5\lambda_m^2\right)\left(1+\mu^2\lambda_m^2\right), m_{23}=\left(I_2+I_7\lambda_m^2\right)\left(1+\mu^2\lambda_m^2\right),$$

$$m_{33}=\left(I_2+I_6\lambda_m^2\right)\left(1+\mu^2\lambda_m^2\right)$$

Appendix B

The Lagrange and Hermite shape functions used herein are as follows:

$$N_0=\frac{1-\xi}{2}, N_1=\frac{1+\xi}{2}, N_2=\frac{1}{2}\sqrt{\frac{3}{2}}\left(\xi^2-1\right), N_3=\frac{1}{2}\sqrt{\frac{5}{2}}\left(\xi^2-1\right)\xi,$$

$$N_3=\frac{1}{8}\sqrt{\frac{7}{2}}\left(\xi^2-1\right)\left(5\xi^2-1\right), N_3=\frac{1}{8}\sqrt{\frac{9}{2}}\left(\xi^2-1\right)\left(7\xi^2-3\right)\xi$$

$$H_0=\frac{1}{2}-\frac{3}{4}\xi+\frac{1}{4}\xi^3, H_1=\frac{l_{\mathrm{e}}}{8}\left(1-\xi-\xi^2+\xi^3\right), H_2=\frac{1}{2}+\frac{3}{4}\xi+\frac{1}{4}\xi^3,$$

$$H_3=\frac{l_{\mathrm{e}}}{8}\left(-1-\xi+\xi^2+\xi^3\right), H_4=\sqrt{\frac{5}{128}}\left(1-\xi^2\right)^2, H_5=\sqrt{\frac{7}{128}}\left(1-\xi^2\right)^2\xi,$$

$$H_6=\frac{1}{6}\sqrt{\frac{9}{128}}\left(1-\xi^2\right)^2\left(1-7\xi^2\right), H_7=\frac{1}{2}\sqrt{\frac{11}{128}}\left(1-\xi^2\right)^2\left(3\xi^2-1\right)\xi,$$

Appendix C

The stiffness and mass matrices of the e^{th} element are as follows:

$$\boldsymbol{K}_e = \boldsymbol{K}_e^{\mathrm{p}} + \boldsymbol{K}_e^{\mathrm{f}} + \boldsymbol{K}_e^{\mathrm{HT1}} + \boldsymbol{K}_e^{\mathrm{Hx}},$$

$$\boldsymbol{K}_e^{\mathrm{p}} = \int_{l_\mathrm{e}} \boldsymbol{B}_\mathrm{m}^\mathrm{T} \boldsymbol{Q} \boldsymbol{B}_\mathrm{m} \mathrm{d}x; \quad \boldsymbol{K}_e^{\mathrm{f}} = \int_{l_\mathrm{e}} \left(k_\mathrm{w} \boldsymbol{B}_\mathrm{w}^\mathrm{T} \boldsymbol{B}_\mathrm{w} + \left(\mu^2 k_\mathrm{w} + k_\mathrm{s} \right) \boldsymbol{B}_{\mathrm{w},x}^\mathrm{T} \boldsymbol{B}_{\mathrm{w},x} + \mu^2 k_\mathrm{s} \boldsymbol{B}_{\mathrm{w},xx}^\mathrm{T} \boldsymbol{B}_{xx} \right) \mathrm{d}x,;$$

$$\boldsymbol{K}_e^{\mathrm{HT1}} = \int_{l_\mathrm{e}} f^{\mathrm{HT}} \left(\boldsymbol{B}_{\mathrm{w},x}^\mathrm{T} \boldsymbol{B}_{\mathrm{w},x} + \mu^2 \boldsymbol{B}_{\mathrm{w},xx}^\mathrm{T} \boldsymbol{B}_{\mathrm{w},xx} \right) \mathrm{d}x,;$$

$$\boldsymbol{K}_e^{\mathrm{Hx}} = \int_{l_\mathrm{e}} \eta h H_x \left(\boldsymbol{B}_{\mathrm{w},x}^\mathrm{T} \boldsymbol{B}_{\mathrm{w},x} + \mu^2 \boldsymbol{B}_{\mathrm{w},xx}^\mathrm{T} \boldsymbol{B}_{\mathrm{w},xx} \right) \mathrm{d}x,$$

$$\boldsymbol{M}_\mathrm{e} = \int_{l_\mathrm{e}} \left(\boldsymbol{N}_\mathrm{ma}^\mathrm{T} \boldsymbol{H}_\mathrm{ma} \boldsymbol{N}_\mathrm{ma} + \mu^2 \boldsymbol{N}_{\mathrm{ma},x}^\mathrm{T} \boldsymbol{H}_\mathrm{ma} \boldsymbol{N}_{\mathrm{ma},x} \right) \mathrm{d}x,.$$

Here,

$$\boldsymbol{B}_\mathrm{m} = \left[\boldsymbol{B}^1; \boldsymbol{B}^2; \boldsymbol{B}^3; \boldsymbol{B}^\mathrm{s} \right]; \ \boldsymbol{B}^1 = \boldsymbol{N}_{,x}^\mathrm{u}; \ \boldsymbol{B}^2 = -\boldsymbol{Q}_{,xx}^\mathrm{b}; \ \boldsymbol{B}^3 = -\boldsymbol{Q}_{,xx}^\mathrm{s}; \boldsymbol{B}^\mathrm{s} = \boldsymbol{Q}_{,x}^\mathrm{s};$$

$$\boldsymbol{N}_\mathrm{ma} = \left[\boldsymbol{N}^\mathrm{u}; \boldsymbol{Q}^\mathrm{b}; \boldsymbol{Q}^\mathrm{s} \right]; \boldsymbol{B}_\mathrm{w} = \boldsymbol{Q}^\mathrm{b} + \boldsymbol{Q}^\mathrm{s},$$

where $(,x)$ and $(,xx)$ represent the first and second partial derivatives with respect to the x-coordinates, respectively.